CHAPTER 5

Shear Strengths of Soil and Municipal Solid Waste

A key step in analyses of soil slope stability is measuring or estimating the strengths of the soils. Meaningful analyses can be performed only if the shear strengths used are appropriate for the soils and for the particular conditions analyzed. Much has been learned about the shear strength of soils within the past 60 years, often from surprising and unpleasant experience with the stability of slopes, and many useful research studies of soil strength have been performed. The amount of information that has been amassed on soil strengths is very large. The following discussion focuses on the principles that govern soil strength, the issues that are of the greatest general importance in evaluating strength, and strength correlations that have been found useful in practice. The purpose is to provide information that will establish a useful framework and a point of beginning for detailed studies of the shear strengths of soils at particular sites.

GRANULAR MATERIALS

The strength characteristics of all types of granular materials (sands, gravels, and rockfills) are similar in many respects. Because the permeabilities of these materials are high, they are usually fully drained in the field, as discussed in Chapter 3. They are cohesionless: The particles do not adhere to one another, and their effective stress shear strength envelopes pass through the origin of the Mohr stress diagram.

The shear strength of these materials can be characterized by the equation

$$s = \sigma' \tan \phi' \tag{5.1}$$

where s is the shear strength, σ' the effective normal stress on the failure plane, and ϕ' the effective stress angle of internal friction. Measuring or estimating the drained strengths of these materials involves determining or estimating appropriate values of ϕ'.

The most important factors governing values of ϕ' for granular soils are density, confining pressure, grain-size distribution, strain boundary conditions, and the factors that control the amount of particle breakage during shear, such as the types of mineral and the sizes and shapes of particles.

Curvature of Strength Envelope

Mohr's circles of stress at failure for four triaxial tests on the Oroville Dam shell material are shown in Figure 5.1. Because this material is cohesionless, the Mohr–Coulomb strength envelope passes through the origin of stresses, and the relationship between strength and effective stress on the failure plane can be expressed by Eq. (5.1).

A *secant value* of ϕ' can be determined for each of the four triaxial tests. This value corresponds to a linear failure envelope going through the origin and passing tangent to the circle of stress at failure for the particular test, as shown in Figure 5.1. The dashed line in Figure 5.1 is the linear strength envelope for the test with the highest confining pressure. The secant value of ϕ' for an individual test is calculated as

$$\phi' = 2\left[\left(\tan^{-1}\sqrt{\frac{\sigma'_{1f}}{\sigma'_{3f}}}\right) - 45^\circ\right] \tag{5.2}$$

where σ'_{1f} and σ'_{3f} are the major and minor principal stresses at failure. Secant values of ϕ' for the tests on Oroville Dam shell material shown in Figure 5.1 are given in Table 5.1, and the envelope for $\sigma'_3 = 4480$ kPa is shown in Figure 5.1.

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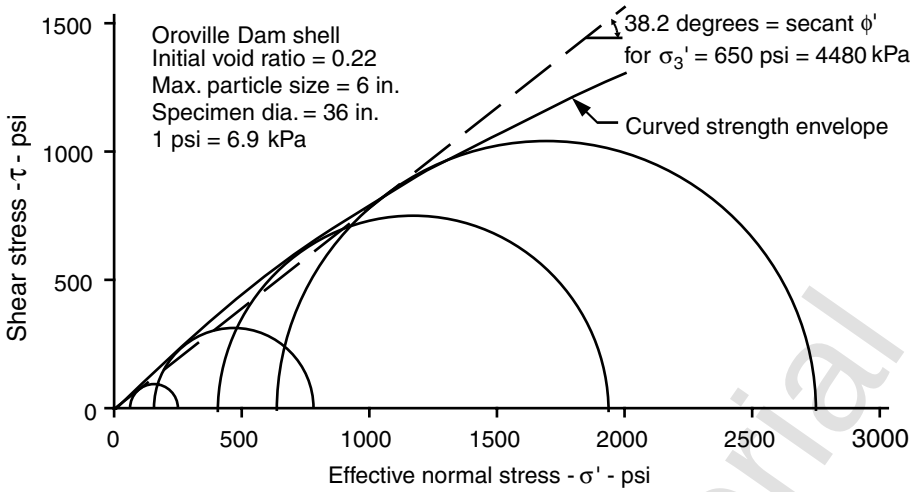


Figure 5.1 Mohr's circles of shear stress at failure and failure envelope for triaxial tests on Oroville Dam shell material. (Data from Marachi et al., 1969.)

Table 5.1 Stresses at Failure and Secant Values of ϕ' for Oroville Dam Shell Material

Test	σ_3' (kPa)	σ_1' (kPa)	ϕ' (deg)
1	210	1,330	46.8
2	970	5,200	43.4
3	2,900	13,200	39.8
4	4,480	19,100	38.2

The curvature of the envelope and the decrease in the secant value of ϕ' as the confining pressure increases are due to increased particle breakage as the confining pressure increases. At higher pressures the interparticle contact forces are larger. The greater these forces, the more likely it is that particles will be broken during shear rather than remaining intact and sliding or rolling over neighboring particles as the material is loaded. When particles break instead of rolling or sliding, it is because breaking requires less energy, and because the mechanism of deformation is changing as the pressures increase, the shearing resistance does not increase in exact proportion to the confining pressure. Even though the Oroville Dam shell material consists of hard amphibolite particles, there is significant particle breakage at higher pressures.

As a result of particle breakage effects, strength envelopes for all granular materials are curved. The envelope does pass through the origin, but the secant value of ϕ' decreases as confining pressure increases. Secant values of ϕ' for soils with curved envelopes can be characterized using two parameters, ϕ_0 and $\Delta\phi$:

$$\phi' = \phi_0 - \Delta\phi \log_{10} \frac{\sigma_3'}{p_a} \tag{5.3}$$

where ϕ' is the secant effective stress angle of internal friction, ϕ_0 the value of ϕ' for $\sigma_3' = 1$ atm, $\Delta\phi$ the reduction in ϕ' for a 10-fold increase in confining pressure, σ_3' the confining pressure, and p_a = atmospheric pressure. This relationship between ϕ' and σ_3' is shown in Figure 5.2*a*. The variation of ϕ' with σ_3' for the Oroville Dam shell material is shown in Figure 5.2*b*.

Values of ϕ' should be selected considering the confining pressures involved in the conditions being analyzed. Some slope stability computer programs have provisions for using curved failure envelopes, which is an effective means of representing variations of ϕ' with confining pressure. Alternatively, different values of ϕ' can be used for the same material, with higher values of ϕ' in areas where pressures are low and lower values of ϕ' in areas where pressures are high. In many cases, sufficient accuracy can be achieved by using a single value of ϕ' based on the average confining pressure.

Effect of Density

Density has an important effect on the strengths of granular materials. Values of ϕ' increase with density. For some materials the value of ϕ_0 increases by 15° or more as the density varies from the loosest to the densest state. Values of $\Delta\phi$ also increase with density, varying from zero for very loose materials to 10° or more for the same materials in a very dense state. An example is shown in Figure 5.3 for Sacramento River sand, a uniform fine sand composed predominantly of

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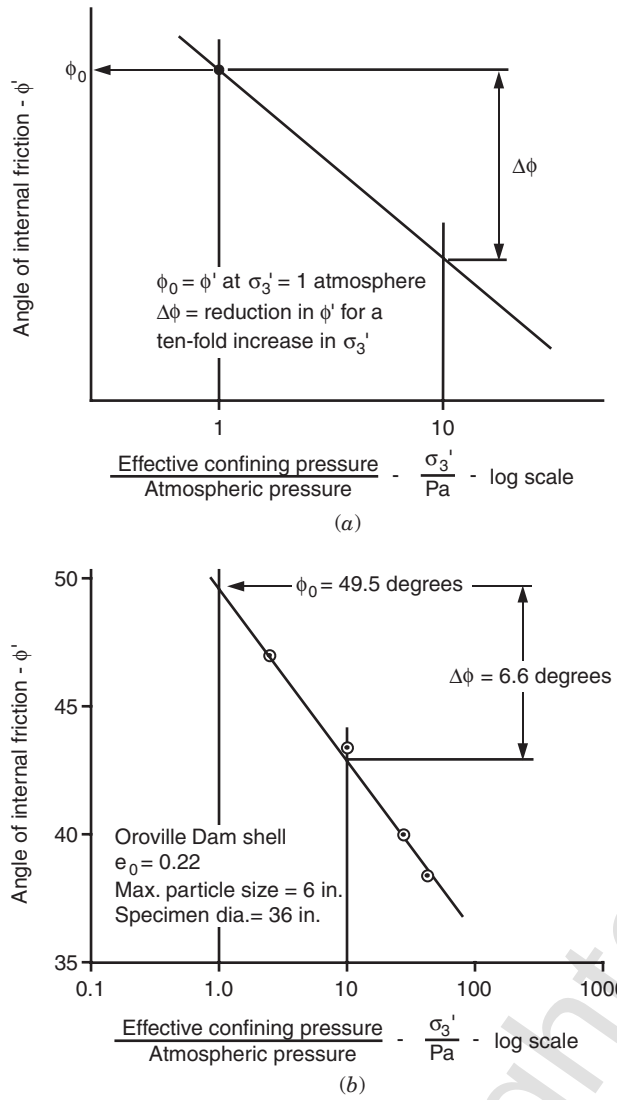


Figure 5.2 Effect of confining pressure on ϕ': (*a*) relationship between ϕ' and σ_3'; (*b*) variation of ϕ' and σ_3' for Oroville Dam shell.

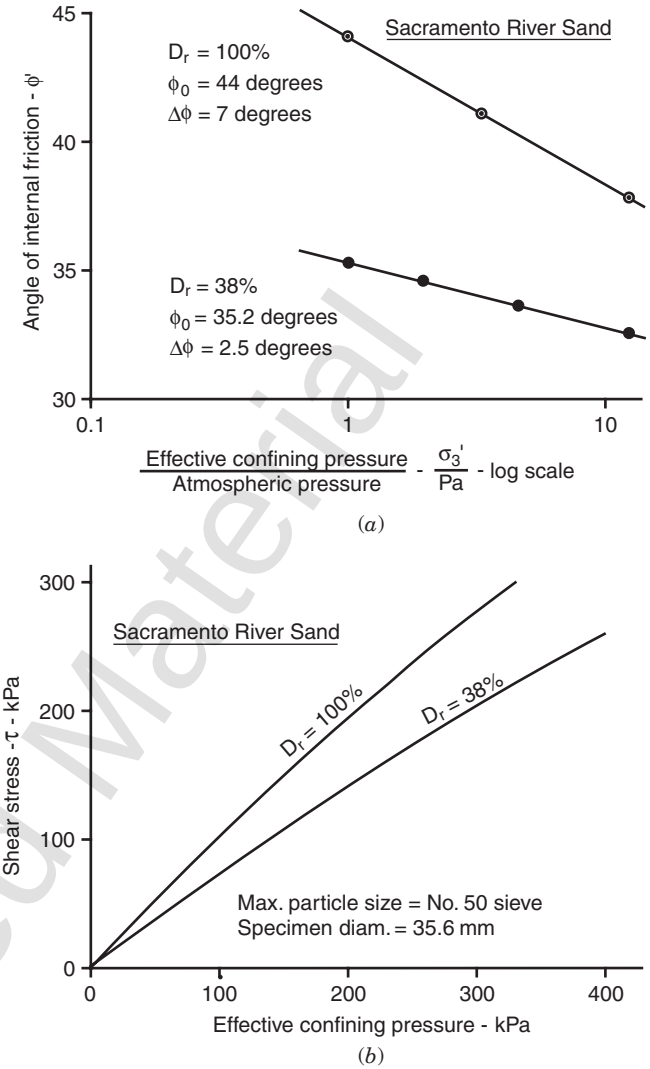


Figure 5.3 Effect of density on strength of Sacramento River sand: (*a*) variations of ϕ' with confining pressure; (*b*) strength envelopes. (Data from Lee and Seed, 1967.)

feldspar and quartz particles. At a confining pressure of 1 atm, ϕ_0 increases from 35° for D_r = 38% to 44° for D_r = 100%. The value of $\Delta\phi$ increases from 2.5° for D_r = 38% to 7° for D_r = 100%

Effect of Gradation

All other things being equal, values of ϕ' are higher for well-graded granular soils such as the Oroville Dam shell material (Figures 5.1 and 5.2) than for uniformly graded soils such as Sacramento River sand (Figure 5.3). In well-graded soils, smaller particles fill gaps between larger particles, and as a result it is possible to form a denser packing that offers greater resistance to shear. Well-graded materials are subject to segregation of particle sizes during fill placement and may form fills that are stratified, with alternating coarser and finer layers unless care is taken to ensure that segregation does not occur.

Plane Strain Effects

Most laboratory strength tests are performed using triaxial equipment, where a circular cylindrical test specimen is loaded axially and deforms with radial symmetry. In contrast, the deformations for many field conditions are close to plane strain. In plane strain, all displacements are parallel to one plane. In the field, this is usually the vertical plane. Strains and displace-

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ments perpendicular to this plane are zero. For example, in a long embankment, symmetry requires that all displacements are in vertical planes perpendicular to the longitudinal axis of the embankment.

The value of ϕ' for plane strain conditions (ϕ'_{ps}) is higher than the value for triaxial conditions (ϕ'_t). Becker et al. (1972) found that the value of ϕ'_{ps} was 1 to 6° larger than the value of ϕ'_t for the same material at the same density, tested at the same confining pressure. The difference was greatest for dense materials tested at low pressures. For confining pressures below 100 psi (690 kPa), they found that ϕ'_{ps} was 3 to 6° larger than ϕ'_t.

Although there may be a significant difference between values of ϕ' measured in triaxial tests and the values most appropriate for conditions close to plane strain, this difference is usually ignored. It is conservative to ignore the difference and use triaxial values of ϕ' for plane strain conditions. This conventional practice provides an intrinsic additional safety margin for situations where the strain boundary conditions are close to plane strain.

Strengths of Compacted Granular Materials

When cohesionless materials are used to construct fills, it is normal to specify the method of compaction or

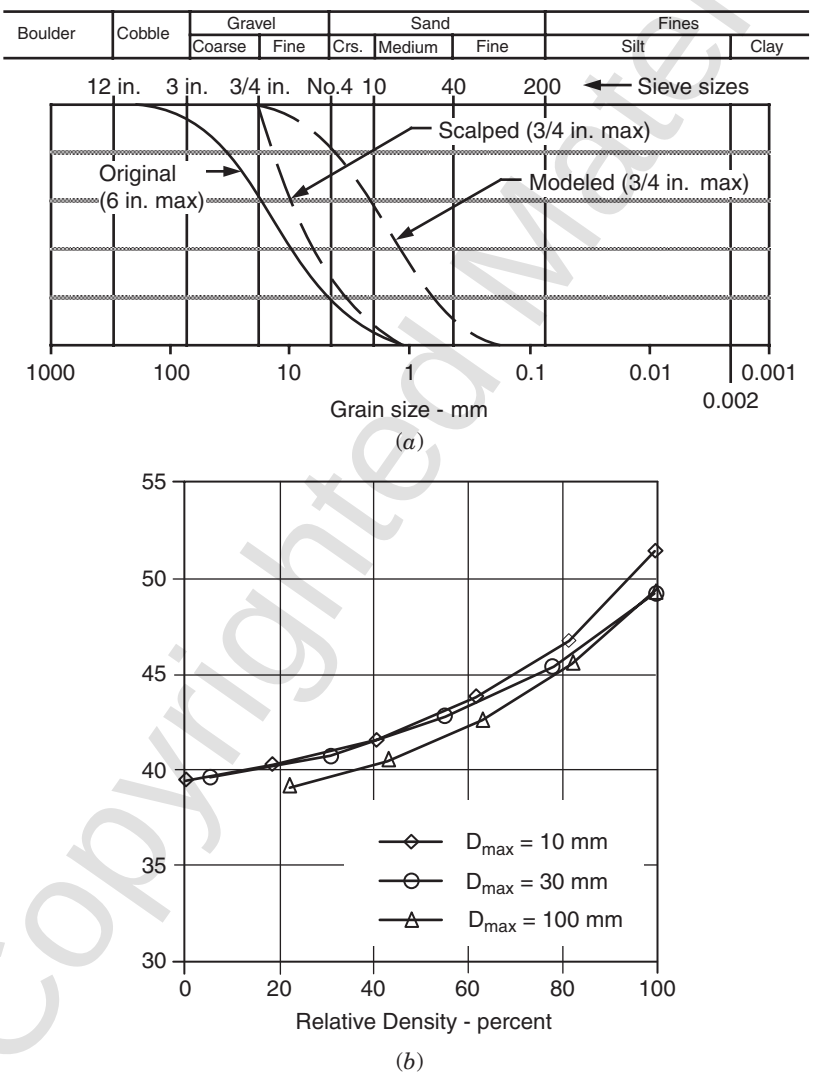


Figure 5.4 Modeling and scalping grain size curves and friction angles for scalped material: (*a*) grain-size curves for original, modeled, and scalped cobbely sandy gravel; (*b*) friction angles for scalped specimens of Goschenalp Dam rockfill. (After Zeller and Wullimann, 1957.)

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the minimum acceptable density. Angles of internal friction for sands, gravels, and rockfills are strongly affected by density, and controlling the density of a fill is thus an effective way of ensuring that the fill will have the desired strength.

Minimum test specimen size. For the design of major structures such as dams, triaxial tests performed on specimens compacted to the anticipated field density are frequently used to determine values of ϕ'. The diameter of the triaxial test specimens should be at least six times the size of the largest soil particle, which can present problems for testing materials that contain large particles. The largest triaxial test equipment available in most soil mechanics laboratories is 100 to 300 mm (4 to 12 in.) in diameter. The largest particle sizes that can be tested with this equipment are thus about 16 to 50 mm (0.67 to 2 in.).

Modeling grain size curves and scalping. When soils with particles larger than one-sixth the triaxial specimen diameter are tested, particles that are too large must be removed. Becker et al. (1972) prepared test specimens with *modeled* grain-size curves. The curves for the modeled materials were parallel to the curve for the original material, as shown in Figure 5.4*a*. It was found that the strengths of the model materials were essentially the same as the strengths of the original materials, provided that the test specimens were prepared at the same relative density, D_r:

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times 100\% \qquad (5.4)$$

where D_r is the relative density, $e_{\max}$ the maximum void ratio, e the void ratio, and $e_{\min}$ the minimum void ratio.

Becker et al. (1972) found that removing large particles changed the maximum and minimum void ratios of the material, and as a result, *the same relative density was not the same void ratio for the original and model materials.* The grain-size modeling technique used by Becker et al. (1972) can be difficult to use for practical purposes. When a significant quantity of coarse material has to be removed, there may not be enough fine material available to develop the model grain-size curve. An easier technique is *scalping,* where the large sizes are not replaced with smaller sizes. A scalped gradation is shown in Figure 5.4*a*.

The data in Figure 5.4*b* show that the value of ϕ' for scalped test specimens is essentially the same as for the original material, *provided that all specimens are prepared at the same relative density*. Again, *the same relative density will not be the same void ratio for the original and scalped materials.*

Controlling field densities. Using relative density to control the densities of laboratory test specimens does not imply that it is necessary to use relative density for control of density in the field during construction. Controlling the density of granular fills in the field using relative density has been found to be difficult, especially when the fill material contains large particles. Specifications based on method of compaction, or on relative compaction, can be used for field control, even though relative density may be used in connection with laboratory tests.

Strengths of Natural Deposits of Granular Materials

It is not possible to obtain undisturbed samples of granular materials, except by exotic procedures such as freezing and coring the ground. In most cases friction angles for natural deposits of granular materials are estimated using the results of in situ tests such as the standard penetration test (SPT) or the cone penetration test (CPT). Correlations that can be used to interpret values of ϕ' from in situ tests are discussed below.

Strength correlations. Many useful correlations have been developed that can be used to estimate the strengths of sands and gravels based on correlations with relative densities or the results of in situ tests. The earliest correlations were developed before the influence of confining pressure on ϕ' was well understood. More recent correlations take confining pressure into account by correlating both ϕ_0 and $\Delta\phi$ with relative density or by including overburden pressure in correlations between ϕ' and the results of in situ tests.

Table 5.2 relates values of ϕ_0 and $\Delta\phi$ to relative density values for well-graded sands and gravels, poorly graded sands and gravels, and silty sands. Figures 5.5 and 5.6 can be used to estimate in situ relative density based on SPT blow count or CPT cone resistance. Values of relative density estimated using Figure 5.5 or 5.6 can be used together with Table 5.2 to estimate values of ϕ_0 and $\Delta\phi$ for natural deposits.

Figures 5.7 and 5.8 relate values of ϕ' to overburden pressure and SPT blow count or CPT cone resistance. Figure 5.9 relates ϕ' to relative density for sands. The values of ϕ' in Figure 5.9 correspond to confining pressures of about 1 atm, and are close to the values of ϕ_0 listed in Table 5.2. Tables 5.3 and 5.4 relate values of ϕ' to SPT blow count and CPT cone resistance. The correlations are easy to use, but they do not take the effect of confining pressure into account.

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Recapitulation

- The drained shear strengths of sands, gravels, and rockfill materials can be expressed as $s = \sigma' \tan \phi'$.
- Values of ϕ' for these materials are controlled by density, gradation, and confining pressure.
- The variation of ϕ' with confining pressure can be represented by

$$\phi' = \phi_0 - \Delta\phi \log_{10} \frac{\sigma_3'}{p_a}$$

  where σ_3' is the confining pressure and p_a is atmospheric pressure.
- When large particles are removed to prepare specimens for laboratory tests, the test specimens should be prepared at the same relative density as the original material, not the same void ratio.
- Values of ϕ' for granular materials can be estimated based on the Unified Soil Classification, relative density, and confining pressure.
- Values of ϕ' for granular materials can also be estimated based on results of standard penetration tests or cone penetration tests.

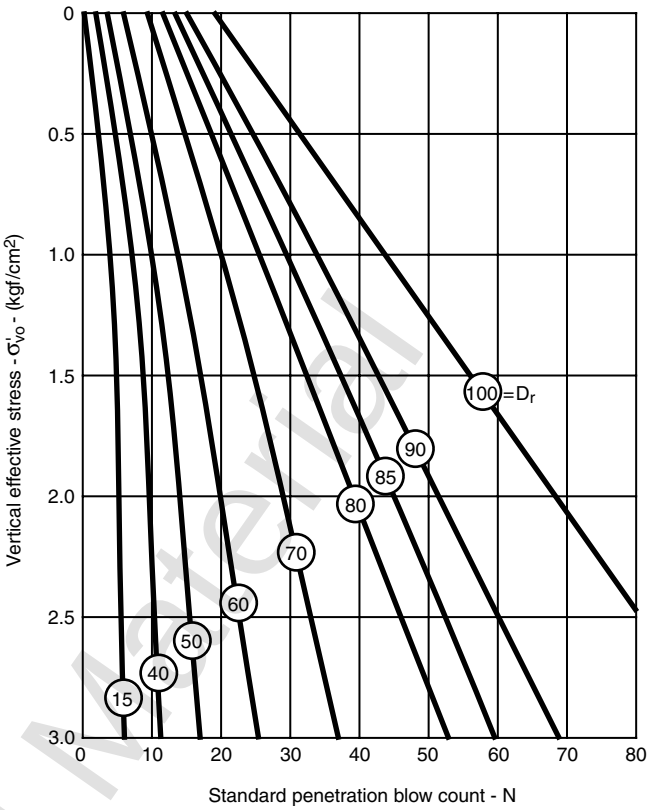


Figure 5.5 Relationship among SPT blow count, overburden pressure, and relative density for sands. (After Gibbs and Holtz, 1957, and U.S. Dept. Interior, 1974.)

Table 5.2 Values of ϕ_0 and $\Delta\phi$ for Sands and Gravels

Unified classification	Standard Proctor RC[a] (%)	Relative density, D_r[b] (%)	ϕ_0[c] (deg)	$\Delta\phi$ (deg)
GW, SW	105	100	46	10
	100	75	43	8
	95	50	40	6
	90	25	37	4
GP, SP	105	100	42	9
	100	75	39	7
	95	50	36	5
	90	25	33	3
SM	100	—	36	8
	95	—	34	6
	90	—	32	4
	85	—	30	2

Source: Wong and Duncan (1974).

[a]RC = relative compaction = $\gamma_d/\gamma_{d\,\max} \times 100\%$.

[b]$D_r = (e_{\max} - e)/e_{\max} - e_{\min}) \times 100\%$.

[c]$\phi' = \phi_0 - \Delta\phi \log_{10} \sigma_3'/p_a$ where p_a is atmospheric pressure.

SILTS

The shear strength of silts in terms of effective stress can be expressed by the Mohr–Coulomb strength criterion as

$$s = c' + \sigma' \tan \phi' \qquad (5.5)$$

where s is the shear strength, c' the effective stress cohesion intercept, and ϕ' the effective stress angle of internal friction.

The behavior of silts has not been studied as extensively and is not as well understood as the behavior of granular materials or clays. Although the strengths of silts are governed by the same principles as the strengths of other soils, the range of their behavior is wide, and sufficient data are not available to anticipate or estimate their properties with the same degree of reliability as is possible in the case of granular soils or clays.

Silts encompass a broad range of behavior, from behavior that is very similar to the behavior of fine sands

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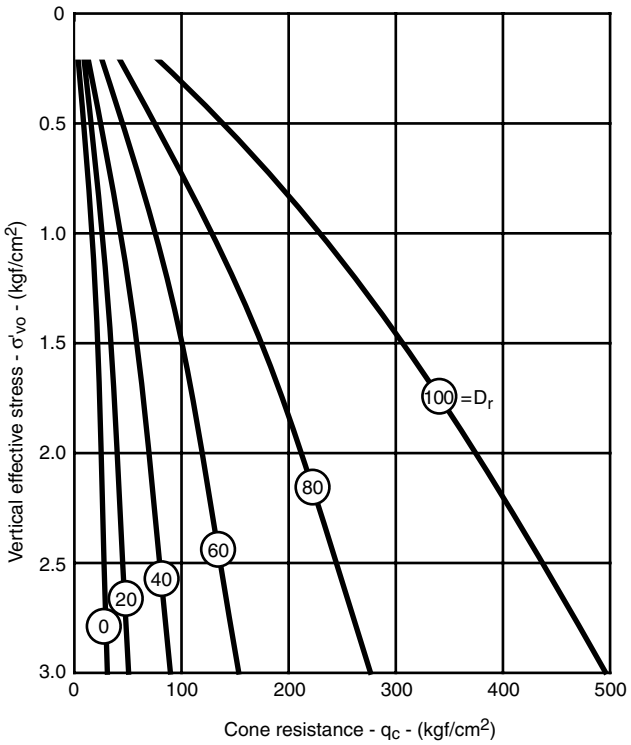


Figure 5.6 Relationship among CPT cone resistance, overburden pressure, and relative density of sands. (After Schmertmann, 1975.)

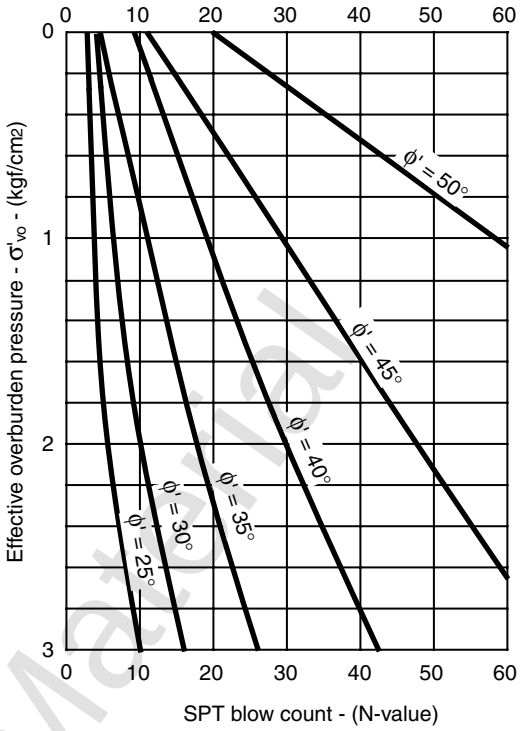


Figure 5.7 Relationship among SPT blow count, overburden pressure, and ϕ' for sands. (After DeMello, 1971, and Schmertmann, 1975.)

at one extreme to behavior that is essentially the same as the behavior of clays at the other extreme. It is useful to consider silts in two distinct categories: nonplastic silts, which behave more like fine sands, and plastic silts, which behave more like clays.

Nonplastic silts, like the silt of which Otter Brook Dam was constructed, behave similarly to fine sands. Nonplastic silts, however, have some unique characteristics, such as lower permeability, that influence their behavior and deserve special consideration.

An example of highly plastic silt is San Francisco Bay mud, which has a liquid limit near 90, a plasiticity index near 45, and classifies as MH (a silt of high plasticity) by the Unified Soil Classification System. San Francisco Bay mud behaves like a normally consolidated clay. The strength characteristics of clays discussed later in this chapter are applicable to materials such as San Francisco Bay mud.

Sample Disturbance

Disturbance during sampling is a serious problem in nonplastic silts. Although they are not highly sensitive by the conventional measure of sensitivity (sensitivity

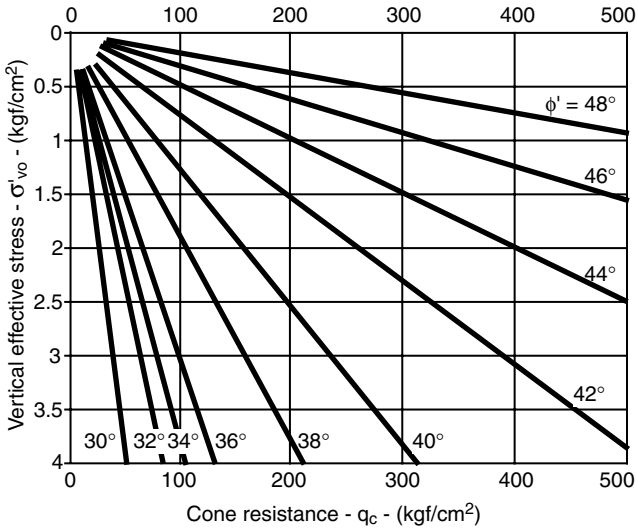


Figure 5.8 Relationship between CPT cone resistance, overburden pressure, and ϕ' for sands. (After Robertson and Campanella, 1983.)

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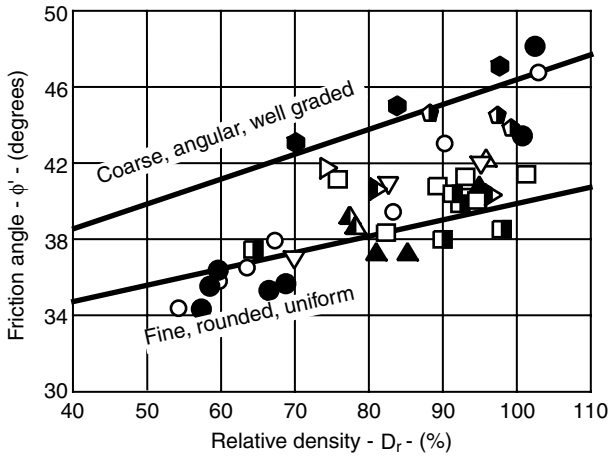


Figure 5.9 Correlation between friction angle and relative density for sands. (Data from Schmertmann, 1975, and Lunne and Kleven, 1982.)

Table 5.3 Relationship Among Relative Density, SPT Blow Count, and Angle of Internal Friction for Clean Sands

State of packing	Relative density, D_r (%)	SPT blow count, N^a (blows/ft)	Angle of internal friction ϕ'^b (deg)
Very loose	< 20	< 4	< 30
Loose	20–40	4–10	30–35
Compact	40–60	10–30	35–40
Dense	60–80	30–50	40–45
Very dense	> 80	> 50	> 45

Source: Meyerhof (1956).

[a] $N = 15 + (N' - 15)/2$ for $N' > 15$ in saturated very fine or silty sand, where N is the blow count corrected for dynamic pore pressure effects during the SPT, and N' is the measured blow count.

[b] Reduce ϕ' by 5° for clayey sand; increase ϕ' by 5° for gravelly sand.

= undisturbed strength/remoulded strength), they are very easily disturbed. In a study of a silt from the Alaskan arctic (Fleming and Duncan, 1990), it was found that disturbance reduced the undrained strengths measured in unconsolidated–undrained tests by as much as 40%, and increased the undrained strengths measured in consolidated–undrained tests by as much as 40%. Although silts can usually be sampled using the same techniques as those used for clays, the quality of samples should not be expected to be as good.

Table 5.4 Correlation Among Relative Density, CPT Cone Resistance, and Angle of Internal Friction for Clean Sands

State of packing	Relative density, D_r (%)	q_c (tons/ft² or kgf/cm²)	ϕ' (deg)
Very loose	< 20	< 20	< 32
Loose	20–40	20–50	32–35
Medium	40–60	50–150	35–38
Dense	60–80	150–250	38–41
Very dense	> 80	250–400	41–45

Source: Meyerhof (1976).

Cavitation

Unlike clays, nonplastic silts almost always tend to dilate when sheared, even if they are normally consolidated. In undrained tests, pore pressures decrease as a result of this tendency to dilate, and pore pressures can become negative. When pore pressures are negative, dissolved air or gas may come out of solution, forming bubbles within test specimens that greatly affect their behavior.

Figure 5.10 shows stress–strain and pore pressure–strain curves for consolidated–undrained triaxial tests on nonplastic silt from the Yazoo River valley. As the specimens were loaded, they tended to dilate, and the pore pressures decreased. As the pore pressures decreased, the effective confining pressures increased. The effective stresses stopped increasing when cavitation occurred, because from that point on the volume of the specimens increased as the cavitation bubbles expanded. The value of the maximum deviator stress for each sample was determined by the initial pore pressure (the back pressure), which determined how much negative change in pore pressure took place before cavitation occurred. The higher the back pressure, the greater was the undrained strength. These effects can be noted in Figure 5.10.

Drained or Undrained Strength?

Values of c_v for nonplastic silts are often in the range 100 to 10,000 cm²/h (1000 to 100,000 ft²/yr). It is often difficult to determine whether silts will be drained or undrained under field loading conditions,

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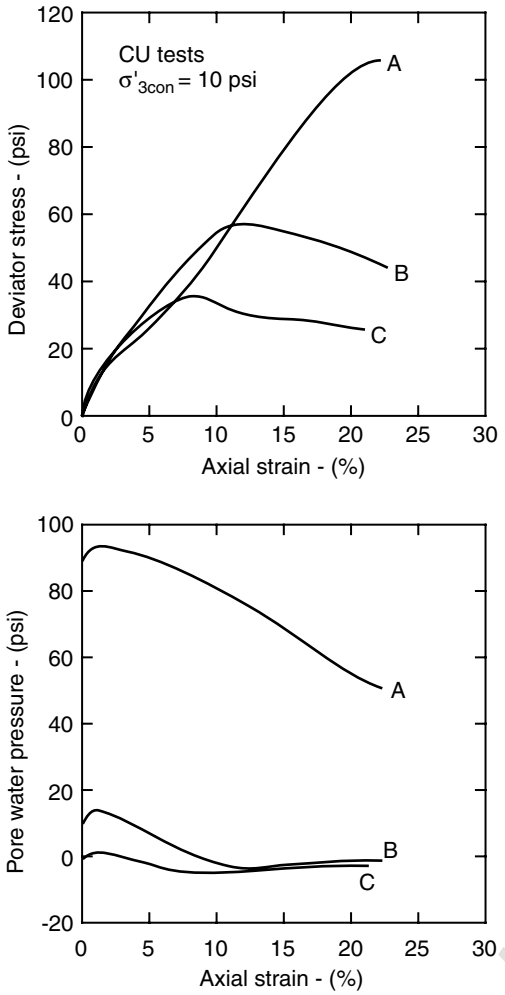


Figure 5.10 Effect of cavitation on undrained strength of reconstituted Yazoo silt. (From Rose, 1994.)

and in many cases it is prudent to consider both possibilities.

Strengths of Compacted Silts

Laboratory test programs for silts to be used as fills can be conducted following the principles that have been established for testing clays. Silts are moisture-sensitive and compaction characteristics are similar to those for clays. Densities can be controlled effectively using relative compaction. Undrained strengths of both plastic and nonplastic silts at the as-compacted condition are strongly influenced by water content.

Nonplastic silts have been used successfully as cores for dams and for other fills. Their behavior during compaction is sensitive to water content, and they become rubbery when compacted close to saturation. In this condition they deform elastically under wheel loads, without failure and without further increase in density. Highly plastic silts, such as San Francisco Bay mud, have also been used as fills, but adjusting the moisture contents of highly plastic materials to achieve the water content and the degree of compaction needed for a high-quality fill is difficult.

Evaluating Strengths of Natural Deposits of Silt

Plastic and nonplastic silts can be sampled using techniques that have been developed for clays, although the quality of the samples is not as good. Disturbance during sampling is a problem for all silts, and care to minimize disturbance effects is important, especially for samples used to measure undrained strengths. Sample disturbance has a much smaller effect on measured values of the effective stress friction angle (ϕ') than it has on undrained strength.

Effective stress failure envelopes for silts can be determined readily using consolidated–undrained triaxial tests with pore pressure measurements, using test specimens trimmed from ''undisturbed'' samples. Drained direct shear tests can also be used. Drainage may occur so slowly in triaxial tests that performing drained triaxial tests may be impractical as a means of measuring drained strengths.

Correlations are not available for making reliable estimates of the undrained strengths of silts, because values of s_u/σ'_{1c} measured for different silts vary widely. A few examples are shown in Table 5.5.

Table 5.5 Values of s_u/σ'_{1c} for Normally Consolidated Alaskan Silts

Test[a]	k_c[b]	s_u/σ'_{1c}	Reference
UU	NA	0.25–0.30	Fleming and Duncan (1990)
UU	NA	0.18	Jamiolkowski et al. (1985)
IC-U	1.0	0.25	Jamiolkowski et al. (1985)
IC-U	1.0	0.30	Jamiolkowski et al. (1985)
IC-U	1.0	0.85–1.0	Fleming and Duncan (1990)
IC-U	1.0	0.30–0.65	Wang and Vivatrat (1982)
AC-U	0.84	0.32	Jamiolkowski et al. (1985)
AC-U	0.59	0.39	Jamiolkowski et al. (1985)
AC-U	0.59	0.26	Jamiolkowski et al. (1985)
AC-U	0.50	0.75	Fleming and Duncan (1990)

[a]UU, unconsolidated undrained triaxial; IC-U, isotropically consolidated undrained triaxial; AC-U, anisotropically consolidated undrained triaxial.

[b]$k_c = \sigma'_{3c}/\sigma'_{1c}$ during consolidation.

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Additional studies will be needed to develop more refined methods of classifying silts and correlations that can be used to make reliable estimates of undrained strengths. Until more information is available, properties of silts should be based on conservative lower-bound estimates, or laboratory tests on the specific material.

Recapitulation

- The behavior of silts has not been studied as extensively, and is not as well understood, as the behavior of granular materials and clays.
- It is often difficult to determine whether silts will be drained or undrained under field loading conditions. In many cases it is prudent to consider both possibilities.
- Silts encompass a broad range of behavior, from fine sands to clays. It is useful to consider silts in two categories: nonplastic silts, which behave more like fine sands, and plastic silts, which behave like clays.
- Disturbance during sampling is a serious problem in nonplastic silts.
- Cavitation may occur during tests on nonplastic silts, forming bubbles within test specimens that greatly affect their behavior.
- Correlations are not available for making reliable estimates of the undrained strengths of silts.
- Laboratory test programs for silts to be used as fills can be conducted following the principles that have been established for testing clays.

CLAYS

Through their complex interactions with water, clays are responsible for a large percentage of problems with slope stability. The strength properties of clays are complex and subject to changes over time through consolidation, swelling, weathering, development of slickensides, and creep. Undrained strengths of clays are important for short-term loading conditions, and drained strengths are important for long-term conditions.

The shear strength of clays in terms of effective stress can be expressed by the Mohr-Coulomb strength criterion as

$$s = c' + \sigma' \tan \phi' \tag{5.6}$$

where s is the shear strength, c' the effective stress cohesion intercept, and ϕ' the effective stress angle of internal friction.

The shear strength of clays in terms of total stress can be expressed as

$$s = c + \sigma \tan \phi \tag{5.7}$$

where c and ϕ are the total stress cohesion intercept and the total stress friction angle.

For saturated clays, ϕ is equal to zero, and the undrained strength can be expressed as

$$s = s_u = c \tag{5.8a}$$

$$\phi = \phi_u = 0 \tag{5.8b}$$

where s_u is the undrained shear strength, independent of total normal stress, and ϕ_u is the total stress friction angle.

Factors Affecting Clay Strength

Low undrained strengths of normally consolidated and moderately overconsolidated clays cause frequent problems with stability of embankments constructed on them. Accurate evaluation of undrained strength, a critical factor in evaluating stability, is difficult because so many factors influence the results of laboratory and in situ tests for clays.

Disturbance. Sample disturbance reduces strengths measured in unconsolidated–undrained (UU) tests in the laboratory. Strengths measured using UU tests may be considerably lower than the undrained strength in situ unless the samples are of high quality. Two procedures have been developed to mitigate disturbance effects (Jamiolkowski et al., 1985):

1. The *recompression technique,* described by Bjerrum (1973), involves consolidating specimens in the laboratory at the same pressures to which they were consolidated in the field. This replaces the field effective stresses with the same effective stresses in the laboratory and squeezes out extra water that the sample may have absorbed as it was sampled, trimmed, and set up in the triaxial cell. This method is used extensively in Norway to evaluate undrained strengths of the sensitive marine clays found there.
2. The *SHANSEP technique,* described by Ladd and Foott (1974) and Ladd et al. (1977), involves consolidating samples to effective stresses that are higher than the in situ stresses, and interpreting the measured strengths in terms of the undrained strength ratio, s_u/σ'_v. Variations of s_u/σ'_v with OCR for six clays, determined from this type of testing, are shown in Figure 5.11. Data of the type shown in Figure 5.11, together with knowledge of the variations of σ'_v and OCR with

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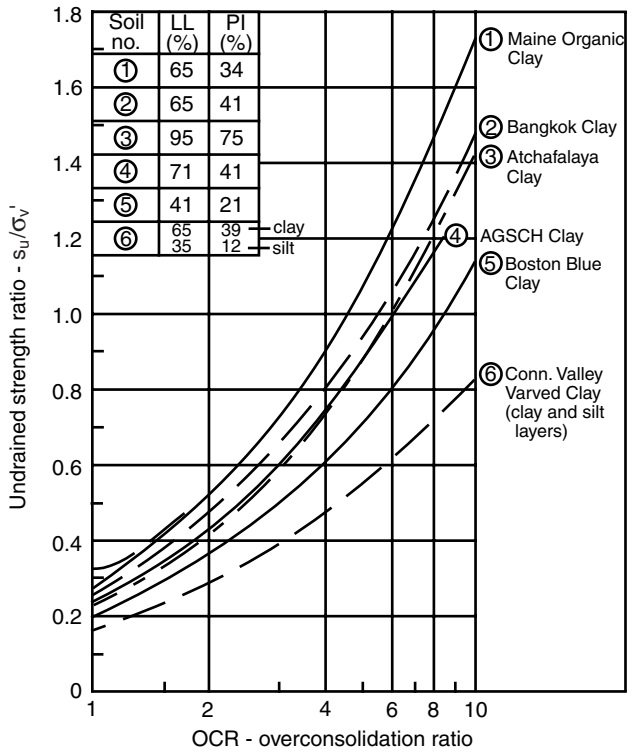


Figure 5.11 Variation of s_u/σ_v' with OCR for clays, measured in ACU direct simple shear tests. (After Ladd et al., 1977.)

depth, can be used to estimate undrained strengths for deposits of normally consolidated and moderately overconsolidated clays.

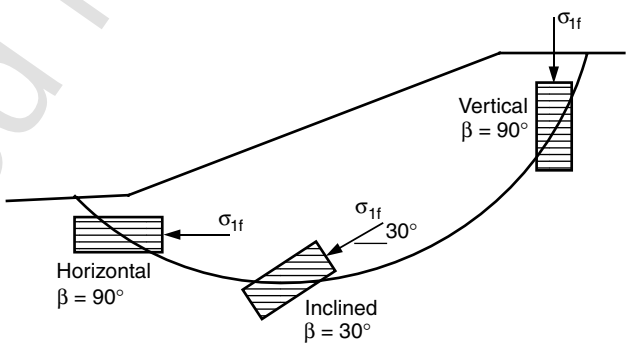
As indicated by Jamiolkowski et al. (1985), both the recompression and the SHANSEP techniques have limitations. The recompression technique is preferable whenever block samples (with very little disturbance) are available. It may lead to undrained strengths that are too low if the clay has a delicate structure that is subject to disturbance as a result of even very small strains (these are called *structured clays*), and it may lead to undrained strengths that are too high if the clay is less sensitive, because reconsolidation results in void ratios in the laboratory that are lower than those in the field. The SHANSEP technique is applicable only to clays without sensitive structure, for which undrained strength increases in direct proportion to the consolidation pressure. It requires detailed knowledge of past and present in situ stress conditions, because the undrained strength profile is constructed using data such as those shown in Figure 5.11, based on knowledge of σ_v' and OCR.

Anisotropy. The undrained strength of clays is *anisotopic;* that is, it varies with the orientation of the failure plane. Anisotropy in clays is due to two effects: inherent anisotropy and stress system-induced anisotropy.

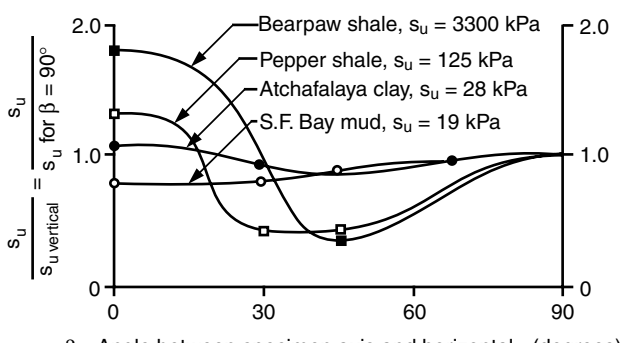
Inherent anisotropy in intact clays results from the fact that plate-shaped clay particles tend to become oriented perpendicular to the major prinicpal strain direction during consolidation, which results in direction-dependent stiffness and strength. Inherent anisotropy in stiff-fissured clays also results from the fact that fissures are planes of weakness.

Stress system-induced anisotropy is due to the fact that the magnitudes of the stresses during consolidation vary depending on the orientation of the planes on which they act, and the magnitudes of the pore pressures induced by undrained loading vary with the orientation of the changes in stress.

The combined result of inherent and stress-induced anisotropy is that the undrained strengths of clays varies with the orientation of the principal stress at failure and with the orientation of the failure plane. Figure 5.12*a* shows orientations of principal stresses and failure planes around a shear surface. Near the top of the shear surface, sometimes called the *active zone,* the



(*a*)



(*b*)

Figure 5.12 Stress orientation at failure, and undrained strength anisotropy of clays and shales: (*a*) stress orientations at failure; (*b*) anisotropy of clays and shales—UU triaxial tests.

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major principal stress at failure is vertical, and the shear surface is oriented about 60° from horizontal. In the middle part of the shear surface, where the shear surface is horizontal, the major principal stress at failure is oriented about 30° from horizontal. At the toe of the slope, sometimes called the *passive zone,* the major principal stress at failure is horizontal, and the shear surface is inclined about 30° past horizontal. As a result of these differences in orientation, the undrained strength ratio (s_u/σ'_v) varies from point to point around the shear surface. Variations of undrained strengths with orientation of the applied stress in the laboratory are shown in Figure 5.12*b* for two normally consolidated clays and two heavily overconsolidated clay shales.

Ideally, laboratory tests to measure the undrained strength of clay would be performed on completely undisturbed plane strain test specimens, tested under unconsolidated–undrained conditions, or consolidated and sheared with stress orientations that simulate those in the field. However, equipment that can apply and reorient stresses to simulate these effects is highly complex and has been used only for research purposes. For practical applications, tests must be performed with equipment that is easier to use, even though it may not replicate all the various aspects of the field conditions.

Triaxial compression (TC) tests, often used to simulate conditions at the top of the slip surface, have been found to result in strengths that are 5 to 10% lower than vertical compression plane strain tests. Triaxial extension (TE) tests, often used to simulate conditions at the bottom of the slip surface, have been found to result in strengths that are significantly less (at least 20% less) than strengths measured in horizontal compression plane strain tests. Direct simple shear (DSS) tests, often used to simulate the condition in the central portion of the shear surface, underestimate the undrained shear strength on the horizontal plane. As a result of these biases, the practice of using TC, TE, and DSS tests to measure the undrained strengths of normally consolidated clays results in strengths that are lower than the strengths that would be measured in ideally oriented plane strain tests.

Strain rate. Laboratory tests involve higher rates of strain than are typical for most field conditions. UU test specimens are loaded to failure in 10 to 20 minutes, and the duration of CU tests is usually 2 or 3 hours. Field vane shear tests are conducted in 15 minutes or less. Loading in the field, on the other hand, typically involves a period of weeks or months. The difference in these loading times is on the order of 1000. Slower loading results in lower undrained shear strengths of saturated clays. As shown in Figure 5.13, the strength of San Francisco Bay mud decreases by about 30% as the time to failure increases from 10 minutes to 1 week. It appears that there is no further decrease in undrained strength for longer times to failure.

In conventional practice, laboratory tests are not corrected for strain rate effects or disturbance effects. Because high strain rates increase strengths measured in UU tests and disturbance reduces them, these effects tend to cancel each other when UU laboratory tests are used to evaluate undrained strengths of natural deposits of clay.

Methods of Evaluating Undrained Strengths of Intact Clays

Alternatives for measuring or estimating undrained strengths of normally consolidated and moderately ov-

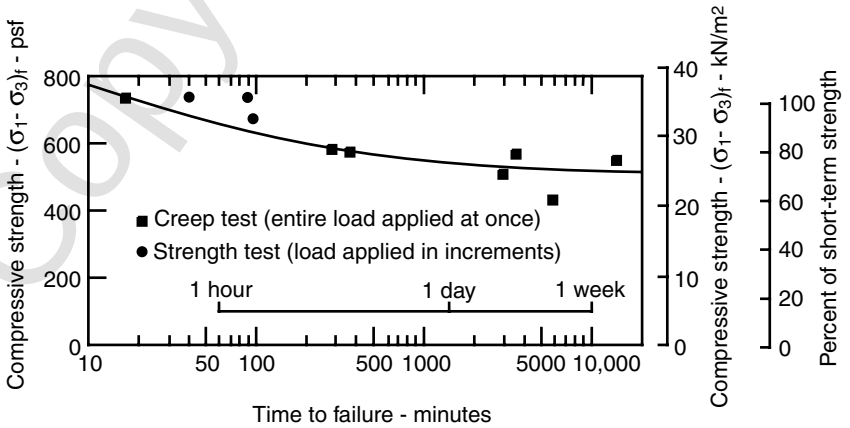


Figure 5.13 Strength loss due to sustained loading.

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erconsolidated clays are summarized in Table 5.6. Samples used to measure strengths of natural deposits of clay should be as nearly undisturbed as possible. Hvorslev (1949) has detailed the requirements for good sampling, which include (1) use of thin-walled tube samplers (wall area no more than about 10% of sample area), (2) a piston inside the tube to minimize strains in the clay as the sample tube is inserted, (3) sealing samples after retrieval to prevent change in water content, and (4) transportation and storage procedures that protect the samples from shock, vibration, and excessive temperature changes. Block samples, carefully trimmed and sealed in moistureproof material, are the best possible types of sample. The consequence of poor sampling is scattered and possibly misleading data. One test on a good sample is better than 10 tests on poor samples.

Field vane shear tests. When the results of field vane shear tests are corrected for strain rate and anisotropy effects, they provide an effective method of measuring the undrained strength of soft and medium clays in situ. Bjerrum (1972) developed correction factors for vane shear tests by comparing field vane (FV) strengths with strengths back-calculated from slope failures. The value of the correction factor, μ, varies with the plasticity index, as shown in Figure 5.14. The data that form the basis for these corrections are rather widely scattered, and vane strengths should not be viewed as precise, even after correction. Nevertheless, the vane shear test avoids many of the problems involved in sampling and laboratory testing and has been found to be a useful tool for measuring the undrained strengths of normally consolidated and moderately overconsolidated clays.

Table 5.6 Methods of Measuring or Estimating the Undrained Strengths of Clays

Procedure	Comments
UU tests on vertical, inclined, and horizontal specimens to determine variation of undrained strength with direction of compression	Relies on counterbalancing effects of disturbance and creep. Empirical method of accounting for anisotropy gives results in agreement with vertical and horizontal plane strain compression tests for San Francisco Bay mud and with field behavior of Pepper shale.
AC-U triaxial compression, triaxial extension, and direct simple shear tests, using the recompression or SHANSEP technique	All three tests give lower undrained strengths than the ideal oriented plane strain tests they approximate. Creep strength loss tends to counterbalance these low strengths.
Field vane shear tests, corrected using empirical correction factors (see Figure 5.14)	Correction accounts for anisotropy and creep strength loss. The data on which the correction factor is based contain considerable scatter.
Cone penetration tests, with an empirical cone factor to evaluate undrained strength (see Figure 5.15)	Empirical cone factors can be determined by comparison with corrected vane strengths or estimated based on published data. Strengths based on CPT results involve at least as much uncertainty as strengths based on vane shear tests.
Standard Penetration Tests, with an empirical factor to evaluate undrained strength (see Figure 5.16)	The Standard Penetration Test is not a sensitive measure of undrained strengths in clays. Strengths based on SPT results involve a great deal of uncertainty.
Use $s_u = [0.23(\mathrm{OCR})^{0.8}]\sigma'_v$	This empirical formula, suggested by Jamiolkowski et al. (1985), reflects the influence of σ'_v (effective overburden pressure) and OCR (overconsolidation ratio), but merely approximates the average of the undrained strengths shown in Figure 5.11. The strengths of particular clays may be higher or lower.
Use $s_u = 0.22\sigma'_p$	This empirical formula, suggested by Mesri (1989) combines the influence of σ'_v and OCR in σ'_p (preconsolidation pressure), resulting in a simpler expression. The degree of approximation is essentially the same as for the formula suggested by Jamiolkowski et al. (1985).

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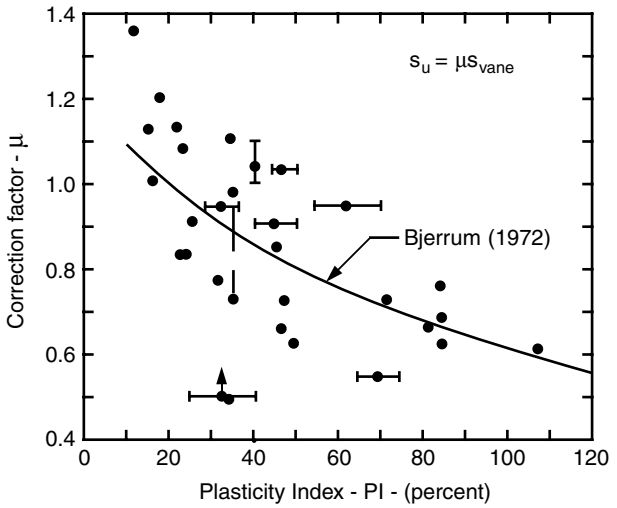


Figure 5.14 Variation of vane shear correction factor and plasticity index. (After Ladd et al., 1977.)

Cone penetration tests. Cone penetration tests (CPTs) are attractive as a means of evaluating undrained strengths of clays in situ because they can be performed quickly and at lower cost than field vane shear tests. The relationship between undrained strength and cone tip resistance is

$$s_u = \frac{q_c - \sigma_{vo}}{N_k^*} \tag{5.9}$$

where s_u is the undrained shear strength, q_c the CPT tip resistance, σ_v the total overburden pressure at the test depth, and N_k^* the cone factor. The units for s_u, q_c, and σ_v in Eq. (5.9) must be the same.

Values of the cone factor N_k^* for a number of different clays are shown in Figure 5.15. These values were developed by comparing *corrected* vane strengths with cone penetration resistance. Therefore Eq. (5.9) provides values of s_u comparable to values determined from field vane shear tests *after correction*. It can be seen that there is little systematic variation of N_k^* with the plasticity index. A value of $N_k^* = 14 \pm 5$ is applicable to clays with any PI value.

A combination of field vane shear and CPT tests can often be used to good advantage to evaluate undrained strengths at soft clay sites. A few vane shear tests are performed close to CPT test locations, and a site-specific value of N_k^* is determined by comparing the results. The cone test is then used for production testing.

Standard Penetration Tests. Undrained strengths can be estimated very crudely based on the results of Standard Penetration Tests. Figure 5.16, which shows

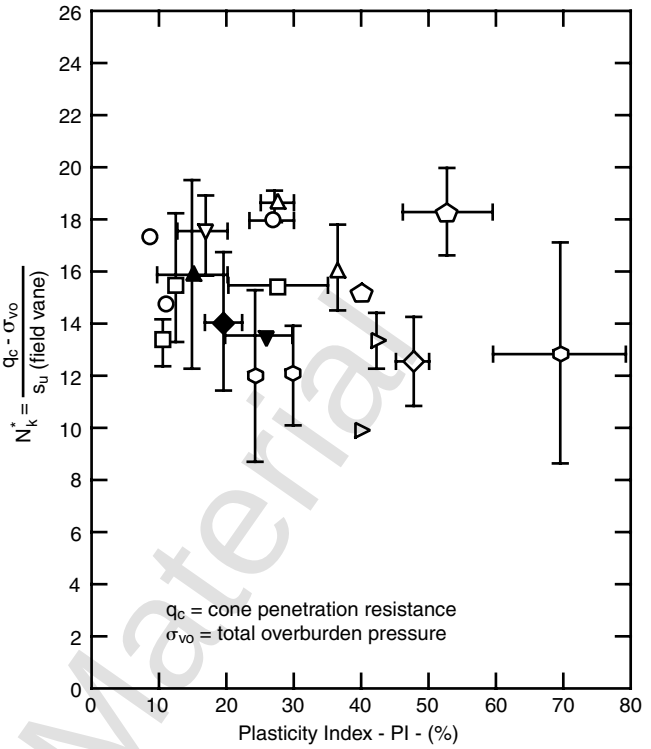


Figure 5.15 Variation of the ratio of net cone resistance ($q_c - \sigma_{vo}$) divided by vane shear strength (s_u vane) with plasticity index for clays. (After Lunne and Klevan, 1982.)

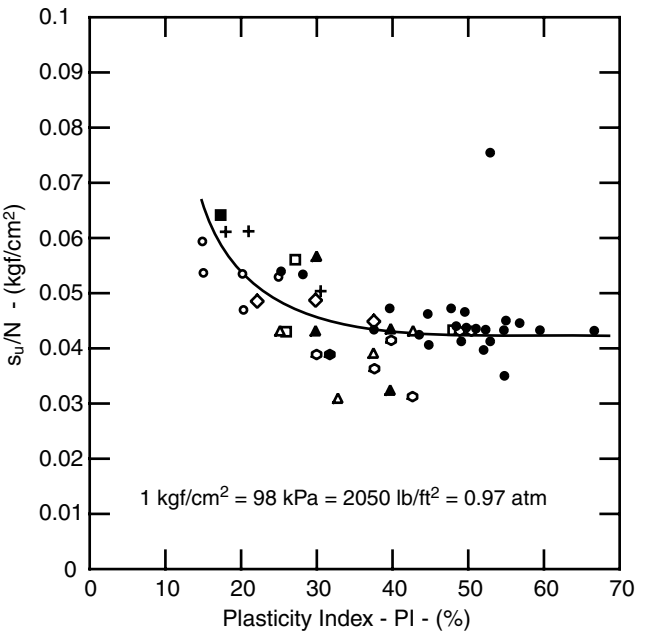


Figure 5.16 Variation of the ratio of undrained shear strength (s_u) divided by SPT blow count (N), with plasticity index for clay. (After Terzaghi et al., 1996.)

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the variation of s_u/N with Plasticity Index, can be used to estimate undrained strength based on SPT blow count. In Figure 5.16 the value of s_u is expressed in kgf/cm^2 (1.0 kgf/cm^2 is equal to 98 kPa, or 1.0 ton per square foot). The Standard Penetration Test is not a sensitive indicator of the undrained strength of clays, and it is not surprising that there is considerable scatter in the correlation shown in Figure 5.16.

Typical Peak Friction Angles for Intact Clays

Tests to measure peak drained strengths of clays include drained direct shear tests and triaxial tests with pore pressure measurements to determine c' and ϕ'. The tests should be performed on undisturbed test specimens. Typical values of ϕ' for normally consolidated clays are given in Table 5.7. Strength envelopes for normally consolidated clays go through the origin of stresses, and $c' = 0$ for these materials.

Stiff-Fissured Clays

Heavily overconsolidated clays are usually stiff, and they usually contain fissures. The term *stiff-fissured clays* is often used to describe them. Terzaghi (1936) pointed out what has since been confirmed by many others—the strengths that can be mobilized in stiff-fissured clays in the field are less than the strength of the same material measured in the laboratory.

Skempton (1964, 1970, 1977, 1985), Bjerrum (1967), and others have shown that this discrepancy is due to swelling and softening that occurs in the field over long periods of time but does not occur in the laboratory within the period of time used to perform laboratory strength tests. A related factor is that fissures, which have an important effect on the strength of the clay in the field, are not properly represented in laboratory samples unless the test specimens are large enough to include a significant number of fissures. Unless the specimen size is several times the average fissure spacing, both drained and undrained strengths measured in laboratory tests will be too high.

Table 5.7 Typical Values of Peak Friction Angle (ϕ') for Normally Consolidated Clays[a]

Plasticity index	ϕ' (deg)
10	33 ± 5
20	31 ± 5
30	29 ± 5
40	27 ± 5
60	24 ± 5
80	22 ± 5

Source: Data from Bjerrum and Simons (1960).
[a] $c' = 0$ for these materials.

Peak, fully softened, and residual strengths of stiff-fissured clays. Skempton (1964, 1970, 1977, 1985) investigated a number of slope failures in the stiff-fissured London clay and developed procedures for evaluating the drained strengths of stiff-fissured clays that have been widely accepted. Figure 5.17 shows stress–displacement curves and strength envelopes for drained direct shear tests on stiff-fissured clays. The undisturbed peak strength is the strength of undisturbed test specimens from the field. The magnitude of the cohesion intercept (c') depends on the size of the test specimens. Generally, the larger the test specimens, the smaller the value of c'. As displacement continues beyond the peak, reached at $\Delta x = 0.1$ to 0.25 in. (3 to 6 mm), the shearing resistance decreases. At displacements of 10 in. (250 mm) or so, the shearing resistance decreases to a residual value. In clays without coarse particles, the decline to residual strength is accompanied by formation of a slickensided surface along the shear plane.

If the same clay is remolded, mixed with enough water to raise its water content to the liquid limit, consolidated in the shear box, and then tested, its peak strength will be lower than the undisturbed peak. The strength after remolding and reconsolidating is shown by the NC (normally consolidated) stress–displacement curve and shear strength envelope. The peak is less pronounced, and the NC strength envelope passes through the origin, with c' equal to zero. As shearing displacement increases, the shearing resistance decreases to the same residual value as in the test on the undisturbed test specimen. The displacement required to reach the residual shearing resistance is again about 10 in. (250 mm).

Studies by Terzaghi (1936), Henkel (1957), Skempton (1964), Bjerrum (1967), and others have shown that factors of safety calculated using undisturbed peak strengths for slopes in stiff-fissured clays are larger than unity for slopes that have failed. It is clear, therefore, that laboratory tests on undisturbed test specimens do not result in strengths that can be used to evaluate the stability of slopes in the field.

Skempton (1970) suggested that this discrepancy is due to the fact that more swelling and softening occurs in the field than in the laboratory. He showed that the NC peak strength, also called the *fully softened strength,* corresponds to strengths back-calculated from *first-time slides,* slides that occur where there is no preexisting slickensided failure surface. Skempton also

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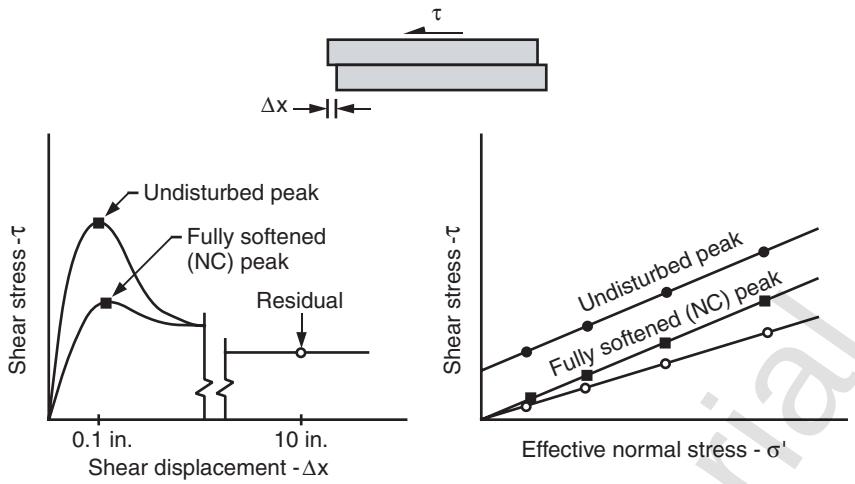


Figure 5.17 Drained shear strength of stiff fissured clay.

showed that once a failure has occurred and a continuous slickensided failure surface has developed, only the residual shear strength is available to resist sliding. Tests to measure fully softened and residual drained strengths of stiff clays can be performed using any representative sample, disturbed or undisturbed, because they are performed on remolded test specimens.

Direct shear tests have been used to measure fully softened and residual strengths. They are more suitable for measuring fully softened strengths because the displacement required to mobilize the fully softened peak strength is small, usually about 0.1 to 0.25 in. (2.5 to 6 mm). Direct shear tests are not so suitable for measuring residual strengths because it is necessary to displace the top of the shear box back and forth to accumulate sufficient displacement to develop a slickensided surface on the shear plane and reduce the shear strength to its residual value. Ring shear tests (Stark and Eid, 1993) are preferable for measuring residual shear strengths because unlimited shear displacement is possible through continuous rotation.

Figures 5.18 and 5.19 show correlations of fully softened friction angle and residual friction angle with liquid limit, clay-size fraction, and effective normal

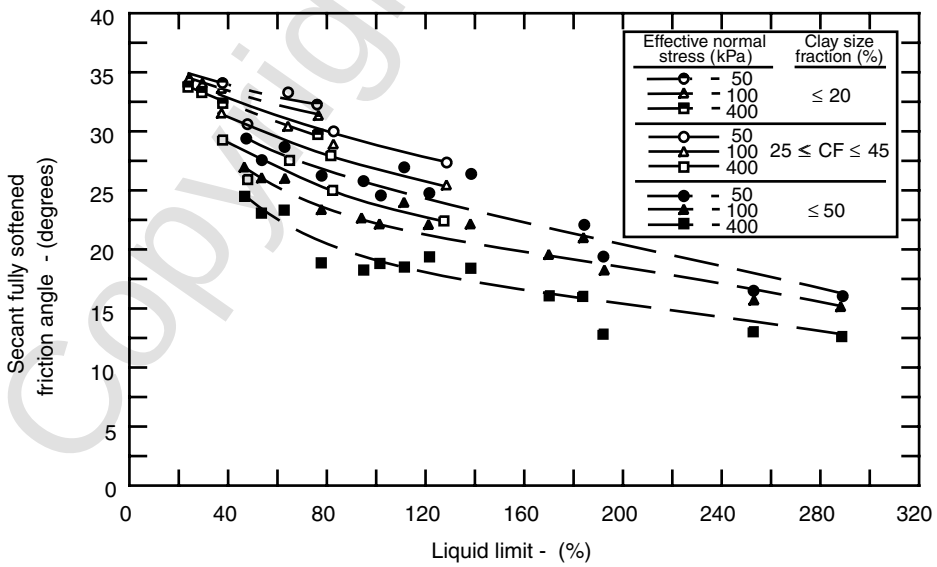


Figure 5.18 Correlation among liquid limit, clay size fraction, and fully softened friction angle. (From Stark and Eid, 1997.)

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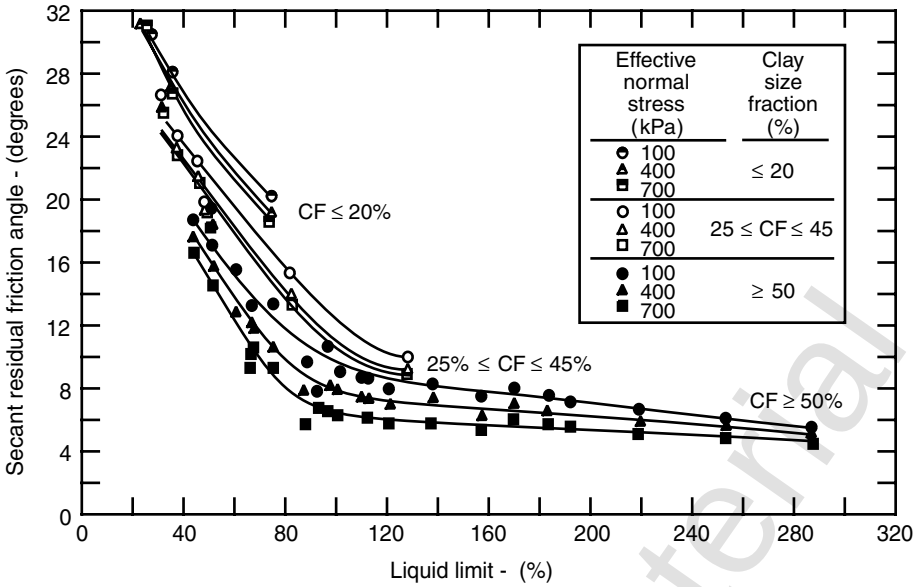


Figure 5.19 Correlation among liquid limit, clay size fraction, and residual friction angle. (From Stark and Eid, 1994.)

stress that were developed by Stark and Eid (1994, 1997). Both fully softened and residual friction angle are fundamental soil properties, and the correlations shown in Figures 5.18 and 5.19 have little scatter. Effective normal stress is a factor because the fully softened and residual strength envelopes are curved, as are the strength envelopes for granular materials. It is thus necessary to represent these strengths using nonlinear relationships between shear strength and normal stress, or to select values of ϕ' that are appropriate for the range of effective stresses in the conditions analyzed.

Undrained strengths of stiff-fissured clays. The undrained strength of stiff-fissured clays is also affected by fissures. Peterson et al. (1957) and Wright and Duncan (1972) showed that the undrained strengths of stiff-fissured clays and shales decreased as test specimen size increased. Small specimens are likely to be intact, with few or no fissures, and therefore stronger than a representative mass of the fissured clay. Heavily overconsolidated stiff fissured clays and shales are also highly anisotropic. As shown in Figure 5.12, inclined specimens of Pepper shale and Bearpaw shale, where

Table 5.8 Typical Peak Drained Strengths for Compacted Cohesive Soils

Unified classification	Relative compaction, RC[a] (%)	Effective stress cohesion, c' (kPa)	Effective stress friction angle, ϕ' (deg)
SM-SC	100	15	33
SC	100	12	31
ML	100	9	32
CL-ML	100	23	32
CL	100	14	28
MH	100	21	25
CH	100	12	19

Source: After U.S. Dept. Interior (1973).

[a]RC, relative compaction by USBR standard method, same energy as the Standard Proctor compaction test.

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failure occurs on horizontal planes, are only 30 to 40% of the strengths of vertical specimens.

Compacted Clays

Compacted clays are used often to construct embankment dams, highway embankments, and fills to support buildings. When compacted well, at suitable water content, clay fills have high strength. Clays are more difficult to compact than are cohesionless fills. It is necessary to maintain their moisture contents during compaction within a narrow range to achieve good compaction, and more equipment passes are needed to produce high-quality fills. High pore pressures can develop in fills that are compacted wet of optimum, and stability during construction can be a problem in wet fills. Long-term stability can also be a problem, particularly with highly plastic clays, which are subject to swell and strength loss over time. It is necessary to consider both short- and long-term stability of compacted fill slopes in clay.

Drained strengths of compacted clays. Values of c' and ϕ' for compacted clays can be measured using consolidated–undrained triaxial tests with pore pressure measurements or drained direct shear tests. The values determined from either type of test are the same for practical purposes. The effective stress strength parameters for compacted clays, measured using samples that have been saturated before testing, are not strongly affected by compaction water content.

Table 5.8 lists typical values of c' and ϕ' for cohesive soils compacted to RC = 100% of the Standard Proctor maximum dry density. As the value of RC decreases below 100%, values of ϕ' remain about the same, and the value of c' decreases. For RC = 90%, values of c' are about half the values shown in Table 5.8.

Recapitulation

- The shear strength of clays in terms of effective stress can be expressed by the Mohr–Coulomb strength criterion as $s = c' + \sigma' \tan \phi'$.
- The shear strength of clays in terms of total stress can be expressed as $s = c + \sigma \tan \phi$.
- For saturated clays, ϕ is zero, and the undrained strength can be expressed as $s = s_u = c$, $\phi = \phi_u = 0$.
- Samples used to measure undrained strengths of normally consolidated and moderately overconsolidated clay should be as nearly undisturbed as possible.
- The strengths that can be mobilized in stiff fissured clays in the field are less than the strength of the same material measured in the laboratory using undisturbed test specimens.
- The normally consolidated peak strength, also called the fully softened strength, corresponds to strengths back calculated from first-time slides.
- Once a failure has occurred and a continuous slickensided failure surface has developed, only the residual shear strength is available to resist sliding.
- Tests to measure fully softened and residual drained strengths of stiff clays can be performed on remolded test specimens.
- Ring shear tests are preferable for measuring residual shear strengths, because unlimited shear displacement is possible through continuous rotation.
- Values of c' and ϕ' for compacted clays can be measured using consolidated–undrained triaxial tests with pore pressure measurements or drained direct shear tests.
- Undrained strengths of compacted clays vary with compaction water content and density and can be measured using UU triaxial tests performed on specimens at their as-compacted water contents and densities.

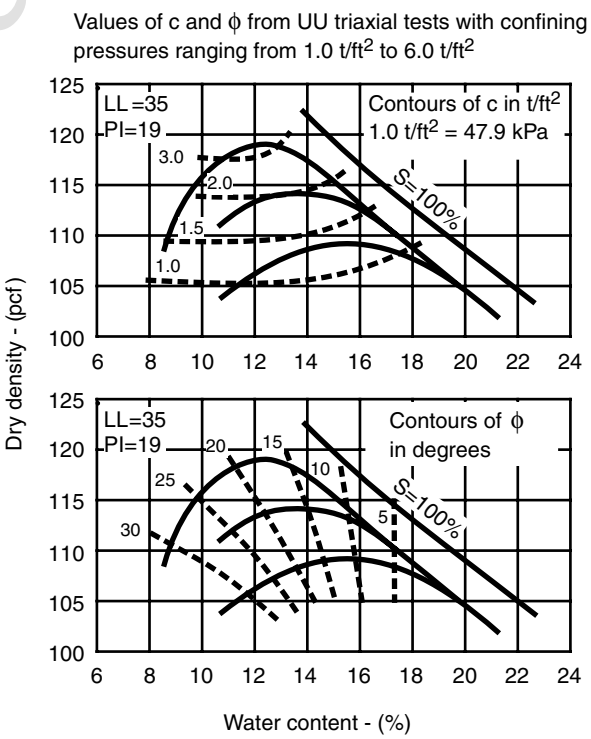


Figure 5.20 Strength parameters for compacted Pittsburgh sandy clay tested under UU test conditions. (From Kulhawy et al., 1969.)

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Undrained strengths of compacted clays. Values of c and ϕ (total stress shear strength parameters) for the as-compacted condition can be determined by performing UU triaxial tests on specimens at their compaction water contents. Undrained strength envelopes for compacted, partially saturated clays tested are curved, as discussed in Chapter 3. Over a given range of stresses, however, a curved strength envelope can be approximated by a straight line and can be characterized in terms of c and ϕ. When this is done, it is especially important that the range of pressures used in the tests correspond to the range of pressures in the field conditions being evaluated. Alternatively, if the computer program used accommodates nonlinear strength envelopes, the strength test data can be represented directly.

Values of total stress c and ϕ for compacted clays vary with compaction water content and density. An example is shown in Figure 5.20 for compacted Pittsburgh sandy clay. The range of confining pressures used in these tests was 1.0 to 6.0 tons/ft^2. The value of c, the total stress cohesion intercept from UU tests, increases with dry density but is not much affected by compaction water content. The value of ϕ, the total stress friction angle, decreases as compaction water content increases, but is not so strongly affected by dry density.

If compacted clays are allowed to age prior to testing, they become stronger, apparently due to thixotropic effects. Therefore, undrained strengths measured using freshly compacted laboratory test specimens provide a conservative estimate of the strength of the fill a few weeks or months after compaction.

MUNICIPAL SOLID WASTE

Waste materials have strengths comparable to the strengths of soils. Strengths of waste materials vary depending on the amounts of soil and sludge in the waste, as compared to the amounts of plastic and other materials that tend to interlock and provide tensile strength (Eid et al., 2000). Larger amounts of materials that interlock increase the strength of the waste. Although solid waste tends to decompose or degrade with time, Kavazanjian (2001) indicates that the strength after degradation is similar to the strength before degradation.

Kavazanjian et al. (1995) used laboratory test data and back analysis of stable slopes to develop the lower-bound strength envelope for municipal solid waste shown in Figure 5.21. The envelope is horizontal with a constant strength $c = 24$ kPa, $\phi = 0$ at normal pressures less than 37 kPa. At pressures greater than 37 kPa, the envelope is inclined at $\phi = 33°$ with $c = 0$.

Eid et al. (2000) used results of large-scale direct shear tests (300 to 1500 mm shear boxes) and back analysis of failed slopes in waste to develop the range of strength envelopes show in Figure 5.22. All three envelopes (lower bound, average, and upper bound) are inclined at $\phi = 35°$. The average envelope shown in Figure 5.22 corresponds to $c = 25$ kPa, and the lowest of the envelopes corresponds to $c = 0$.

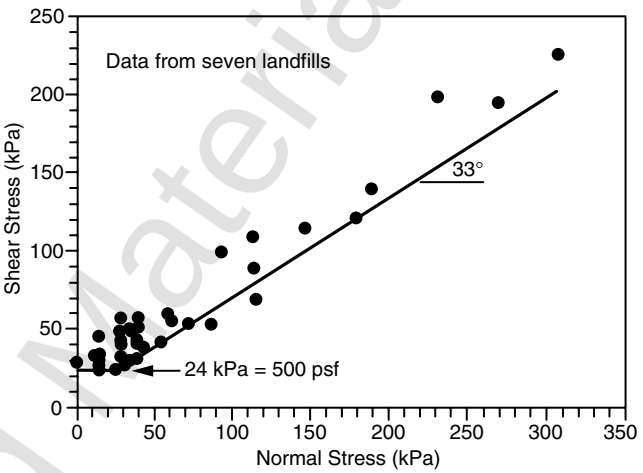


Figure 5.21 Shear strength envelope for municipal solid waste based on large-scale direct shear tests and back analysis of stable slopes. (After Kavazanjian et al., 1995.)

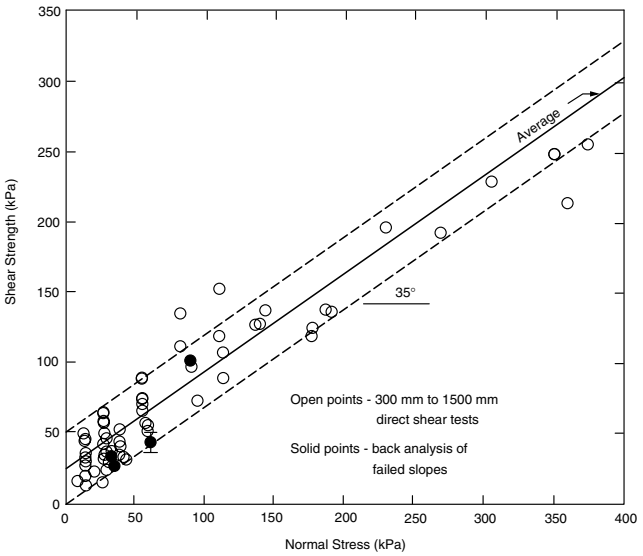


Figure 5.22 Range of shear strength envelopes for municipal solid waste based on large-scale direct shear tests and back analysis of failed slopes. (After Eid et al., 2000.)

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